Sta 111 - Summer II 2017
Probability and Statistical Inference

3. Conditional probability

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Outline

Researchers randomly assigned 72 chronic users of cocaine into three groups: desipramine (antidepressant), lithium (standard treatment for cocaine) and placebo. Results of the study are summarized below.

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

http://www.oswego.edu/~srp/stats/2_way_tbl_1.htm

Marginal probability

What is the probability that a patient relapsed?

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48*48*	24	72*72*

P(relapsed) = $\frac{48}{72} \approx 0.67$

Joint probability

What is the probability that a patient received the antidepressant (desipramine) and relapsed?

	relapse	no relapse	total
desipramine	10 *10*	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72 *72*

P(relapsed and desipramine) = $\frac{10}{72} \approx 0.14$

Conditional probability

Conditional probability

The conditional probability of the outcome of interest A given condition B is calculated as

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

$$P(relapse|desipramine) = \frac{P(relapse\ and\ desipramine)}{P(desipramine)} = \frac{10/72}{24/72} = \frac{10}{24} = 0.42$$

Generically, if $P(A|B) = P(A)$ then the events A and B are said to be *independent*.

- Conceptually: Giving B doesn't tell us anything about A.
- Mathematically: We know that if events A and B are independent, $P(A \text{ and } B) = P(A) \times P(B)$. Then,

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

General multiplication rule

- If two events are independent, their joint probability is simply the product of their probabilities. If the events are not believed to be independent, the joint probability is calculated slightly differently.
- If A and B represent two outcomes or events, then

General multiplication rule

$$P(A \text{ and } B) = P(A|B) \times P(B)$$

Note that this formula is simply the conditional probability formula, rearranged.

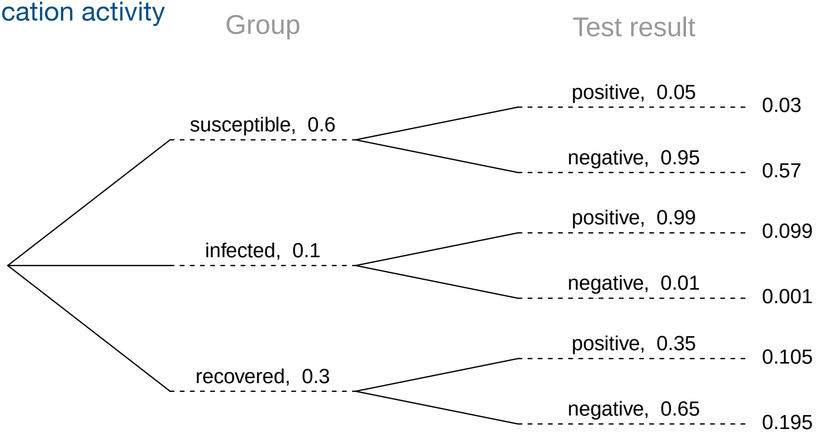
- It is useful to think of A as the outcome of interest and B as the condition.

- Bayes' Theorem is especially useful in cases where you know $P(A \mid B)$, along with some other information, and you are asked for $P(B \mid A)$.

- **Bayes' Theorem**

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$$

Application activity



$$
\begin{aligned}
P(\textit{infected} \mid +) &= \frac{P(+ \mid \textit{infected})P(\textit{infected})}{P(+)} \\
&= \frac{0.99 \times 0.1}{0.6 \times 0.05 + 0.1 \times 0.99 + 0.3 \times 0.35} \approx 0.423
\end{aligned}
$$